

$$6.3. \text{ 1a. } \int 2x(x^2+1)^{23} dx \quad u = x^2+1$$

$$\cancel{2x}(x^2+1)^{23} \frac{du}{2x}$$

$$\int (x^2+1)^{23} du$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2x} = \frac{2x dx}{2x}$$

$$\frac{du}{2x} = dx$$

$$\frac{u^{24}}{24} = \frac{(x^2+1)^{24}}{24} + C$$

$$\int 0 dx = C$$

$$\int 1 dx = x + C$$

$$\int x dx = \frac{x^2}{2} + C$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

1.6 $\int \cos^3 x \sin x \, dx$ $u = \cos x$

$\frac{du}{dx} = -\sin(x)$

$\frac{du}{-\sin x} = dx$

$\int u^3 \sin x \frac{du}{-\sin x}$

$\int -u^3 du = -\frac{u^4}{4} + C$

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$-\frac{(\cos(x))^4}{4} + C$

$$C. \int \frac{1}{\sqrt{x}} \sin(\sqrt{x}) dx$$

$$\int \sin(\sqrt{x}) \frac{1}{\sqrt{x}} 2\sqrt{x} du$$

$$2 \sin(\sqrt{x})$$

$$\int 2 \sin(u) du$$

$$\int 2 \sin(u) =$$

$$-2 \cos(u) \Rightarrow$$

$$(-2 \cos \sqrt{x}) + C$$

$$u = \sqrt{x}$$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{dx}{du}$$

$$\frac{1}{\frac{1}{2\sqrt{x}}} = dx$$

$$\frac{1}{\frac{1}{2\sqrt{x}}} = 2\sqrt{x} du$$

$$\frac{dv}{dx} = 4 \quad dx = \frac{dv}{4} \quad \frac{\int \sec^2(u) dv}{4} = \frac{\tan(u)}{4}$$

$$\frac{\tan(4x+1)}{4} + C$$

$$b) \int y \sqrt{1+2y^2} dy \quad u=1+2y^2 \quad \frac{du}{dy} = 4y \quad dy = \frac{du}{4}$$

$$\int \frac{\sqrt{u} du}{4} = \frac{(u)^{3/2}}{\frac{4 \cdot \frac{3}{2}}{1}} = \frac{(u)^{3/2}}{6} = \frac{(1+2y^2)^{3/2}}{6} + C$$

$$\begin{aligned} \int \sqrt{\sin \pi \theta} \cos \pi \theta d\theta & \quad u = \sin \pi \theta \\ \int \frac{\sqrt{u} \cos \pi \theta du}{\cos \pi \theta (\pi)} &= \frac{\sqrt{u}}{\pi} \quad \frac{u^{3/2}}{\frac{\pi}{3/2}} \\ \frac{2u^{3/2}}{3\pi} &= \frac{2(\sin \pi \theta)^{3/2}}{3\pi} + C \end{aligned}$$

$$d \int (2x+7) (x^2+7x+3)^{4/5} dx \quad u = x^2+7x+3$$

$$\int u^{4/5} du = \frac{u^{9/5}}{\frac{9}{5}} \quad \frac{du}{dx} = 2x+7 \quad du = 2x+7 dx$$

$$\frac{5(x^2+7x+3)^{9/5}}{9} + C$$

$$\underline{3a)} \int \cot x \csc^2 x dx \quad u = \cot x$$
$$\frac{du}{dx} = -\csc^2$$

1d) $\int \frac{3x dx}{\sqrt{4x^2+5}}$ $u = 4x^2 + 5$

$\int \frac{3x dx}{\sqrt{u}}$

$\int \frac{3x \frac{du}{dx}}{\sqrt{u}} \quad \frac{3x du}{dx}$

$\frac{du}{dx} = \frac{8x}{1} \quad dx = \frac{du}{8x}$

$\int \frac{3 \frac{du}{8}}{\sqrt{u}} \quad \int \frac{3 du}{8\sqrt{u}}$

$\frac{3}{8} \int \frac{du}{u^{1/2}}$

$\frac{3}{8} \cdot \frac{u^{1/2+1}}{1/2+1} + C$

$\frac{3}{8} \cdot \frac{u^{3/2}}{3/2} + C$

$\frac{3}{8} \cdot \frac{2}{3} u^{3/2} + C$

$\frac{1}{4} u^{3/2} + C$

$\frac{1}{4} (4x^2+5)^{3/2} + C$

$\frac{1}{4} (4x^2+5)\sqrt{4x^2+5} + C$

$\frac{1}{4} (4x^2+5)^{1/2} + C$

$\frac{1}{4} \sqrt{4x^2+5} + C$